3) $c + \sqrt{c} = 12 + x\sqrt{9+y^2}$

a) si $x=6, y=4 \Rightarrow c+\sqrt{c} = 12 + 6\sqrt{9+4^2}$

$c+\sqrt{c} = 12 + 6(5) \Rightarrow c+\sqrt{c} = 42$

$\therefore \boxed{c = 36}$

b) $\frac{\partial c}{\partial x} = ? \Rightarrow \frac{\partial}{\partial x}(c+\sqrt{c}) = \frac{\partial}{\partial x}(12 + x\sqrt{9+y^2})$

$\frac{\partial c}{\partial x} + \frac{1}{2}c^{-1/2}\cdot\frac{\partial c}{\partial x} = \sqrt{9+y^2}$

$c^{-1/2} = \frac{1}{c^{1/2}} = \frac{1}{\sqrt{c}}$

$c = f(x,y)$

$\frac{\partial c}{\partial x} = \frac{\sqrt{9+y^2}}{\left(1+\frac{1}{2}\cdot\frac{1}{c^{1/2}}\right)}$

$\left.\frac{\partial c}{\partial x}\right|_{(x=6,y=4)} = \frac{\sqrt{9+4^2}}{\left(1+\frac{1}{2}\frac{1}{\sqrt{36}}\right)} = \frac{5}{1+\frac{1}{12}}$

$\frac{\partial c}{\partial y} = ? \Rightarrow \frac{\partial}{\partial y}(c+\sqrt{c}) = \frac{\partial}{\partial y}\left(12 + x(9+y^2)^{1/2}\right)$

$\frac{\partial c}{\partial y} + \frac{1}{2}c^{-1/2}\frac{\partial c}{\partial y} = \frac{1}{\cancel{2}}x(9+y^2)^{-1/2}\cdot(\cancel{2}y)$

$(9+y^2)^{-1/2} = \frac{1}{(9+y^2)^{1/2}} = \frac{1}{\sqrt{9+y^2}}$

$\frac{\partial c}{\partial y}\left(1+\frac{1}{2}\left(\frac{1}{\sqrt{c}}\right)\right) = \frac{x\,y}{(9+y^2)^{1/2}}$

$\frac{\partial c}{\partial y} = \frac{\frac{xy}{\sqrt{(9+y^2)}}}{1+\frac{1}{2\sqrt{c}}}$

4)

$f(x,y) = x^3 - 3xy + y^3$

$f_x = 3x^2 - 3y \Rightarrow 3x^2 - 3y = 0 \Rightarrow 3x^2 = 3y \Rightarrow y = x^2$

$f_y = -3x + 3y^2 \Rightarrow -3x + 3y^2 = 0 \Rightarrow 3y^2 - 3x = 0$

$3(x^2)^2 - 3x = 0$

$3x^4 - 3x = 0$

$3x(x^3-1) = 0$

$\Rightarrow x = 0 \vee x = 1$

Si $x=0 \Rightarrow y=0$ $\therefore (0,0)$

Si $x=1 \Rightarrow y=1$ $\therefore (1,1)$

PUNTOS CRÍTICOS

(5)

$$U = I - C$$

$$U(x,y) = (-0.2x^2 - 0.25y^2 - 0.2xy + 200x + 100y) - (100x + 70y + 4000)$$

$$U(x,y) = -0.2x^2 - 0.25y^2 - 0.2xy + 100x + 30y - 4000$$

$$U_x = -0.4x - 0.2y + 100 \Rightarrow \begin{cases} -0.4x - 0.2y + 100 = 0 \\ -0.5y - 0.2x + 30 = 0 \end{cases}$$

$$U_y = -0.5y - 0.2x + 30$$

$U_{xx} = -0.4$ $\quad U_{xy} = -0.2$ $\quad (x = 275, y = -50)$

$U_{yy} = -0.5$

$$U_{xx} \cdot U_{yy} - (U_{xy})^2$$

$$D(275,-50) = f_{xx} \cdot f_{yy} - (f_{xy})^2$$

$$= (-0.4) \cdot (-0.5) - (-0.2)^2$$

$$= 0.2 - 0.04 = 0.16 > 0$$

$$U_{xx} = f_{xx} = -0.4 < 0$$

$\Rightarrow (275,-50)$ es un punto máximo

$$U(x,y) = -0.2x^2 - 0.25y^2 - 0.2xy + 100x + 30y - 4000$$

$$U(275,-50) = 9000$$

1) $f(x,y) = -x^2y + x^2y^2 + 3x + 5y + 100xy$

$f_x = -2xy + 2xy^2 + 3 + 100y$

$\to f_{xx} = -2y + 2y^2$

$\to f_{xy} = -2x + 4xy + 100$

$f_y = -x^2 + 2x^2y + 5y + 100x$

$\to f_{yx} = -2x + 4xy + 100$

$\to f_{yy} = 2x^2 + 5$

2) $x^2 - 2y + z^2 + x^2yz^2 = 20$

i) $\frac{\partial z}{\partial x} = ?$ $\Rightarrow$ $\frac{\partial}{\partial x}\left(x^2 - 2y + z^2 + \underbrace{x^2}_{f}\underbrace{yz^2}_{g}\right) = \frac{\partial}{\partial x}(20)$

$2x - 0 + 2z\frac{\partial z}{\partial x} + \left(2x\cdot yz^2 + x^2\left(2yz\cdot\frac{\partial z}{\partial x}\right)\right) = 0$

$2x + 2z\frac{\partial z}{\partial x} + 2xyz^2 + 2x^2yz\frac{\partial z}{\partial x} = 0$

$\frac{\partial z}{\partial x}(2z + 2x^2yz) = -2x - 2xyz^2$

$\frac{\partial z}{\partial x} = \frac{-2x - 2xyz^2}{2z + 2x^2yz}$

$(f\cdot g)' = f'g + f\cdot g'$

ii) $\frac{\partial z}{\partial y} = ?$ $\Rightarrow$ $\frac{\partial}{\partial y}\left(x^2 - 2y + z^2 + \underbrace{x^2\cdot y}_{f}\cdot\underbrace{z^2}_{g}\right) = \frac{\partial}{\partial y}(20)$

$-2 + 2z\frac{\partial z}{\partial y} + x^2\cdot z^2 + x^2\cdot y\cdot 2z\frac{\partial z}{\partial y} = 0$

$\frac{\partial z}{\partial y}(2z + 2x^2yz) = 2 - x^2z^2$

$\frac{\partial z}{\partial y} = \frac{2 - x^2\cdot z^2}{2z + 2x^2yz}$